PERIYAR UNIVERSITY

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Salem-636011, Tamilnadu, India.

CENTRE FOR DISTANCE AND ONLINE EDUCATION
(CDOE)

BACHELOR OF SCIENCE – (Computer Science)
SEMESTER - II

ELECTIVE COURSE: NUMERICAL METHODS-PRACTICAL
(Candidates admitted from 2024 onwards)

PERIYAR UNIVERSITY

CENTRE FOR DISTANCE AND ONLINE EDUCATION (CDOE)

B.SC 2024 admission onwards

ELECTIVE – II
NUMERICAL METHODS-PRACTICAL

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SYLLABUS

NUMERICAL METHODS-PRACTICAL

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Unit-I

BISECTION METHOD

AIM

To find the roots of a polynomial equation.

OBJECTIVE

In this section you will learn to

- Separate the interval in which the root of the equation lies

PROCEDURE

Step 1: Assume that $f(x)$ is continuous and it can be algebraic or transcendental.

Step 2: If $f(a)$ and $f(b)$ are of opposite signs, atleast one real root between a and b should exist.

Step 3: Assume that root to be $x_0 = \frac{a+b}{2}$

Step 4: Find the sign of $f(x_0)$. If $f(x_0)$ is negative then the root lies between a and x_0. If $f(x_0)$ is positive then the root lies between x_0 and b.

PROBLEMS

Find the positive root of $x^3 - x = 1$ correct to four decimal places by bisection method.

Solution

Let $f(x) = x^3 - x - 1$

Here $f(0) = -1 = -ve, f(1) = -ve$, $f(2) = 5 = +ve$

Hence the root lies between 1 and 2. Let $I = [1,2]$

Let $x_0 = \frac{1+2}{2} = 1.5$

Now $f(x_0) = f(1.5) = +ve$ and $f(1) = -ve$

Hence the root lies between 1 and 1.5.

Let $x_1 = \frac{1+1.5}{2} = 1.25$

Now $f(x_1) = f(1.25) = -ve$ and $f(1.5) = +ve$

Hence the root lies between 1.25 and 1.5.

Let $x_2 = \frac{1.25+1.5}{2} = 1.375$

Now $f(x_2) = f(1.375) = +ve$

Hence the root lies between 1.25 and 1.375.

Let $x_3 = \frac{1.25+1.375}{2} = 1.3125$

Now $f(x_3) = f(1.3125) = -ve$

Hence the root lies between 1.3125 and 1.375.

Let $x_4 = \frac{1.3125+1.375}{2} = 1.3438$

Now $f(x_4) = f(1.3438) = +ve$

Hence the root lies between 1.3125 and 1.3438.

Let $x_5 = \frac{1.3125+1.3438}{2} = 1.3282$

Now $f(x_5) = f(1.3282) = +ve$

Hence the root lies between 1.3125 and 1.3282.

Let $x_6 = \frac{1.3125+1.3282}{2} = 1.3204$

Now $f(x_6) = f(1.3204) = -ve$

Hence the root lies between 1.3204 and 1.3282.

Let $x_7 = \frac{1.3204+1.3282}{2} = 1.3243$

Now $f(x_7) = f(1.3243) = -ve$

Hence the root lies between 1.3243 and 1.3282.

Let $x_8 = \frac{1.3243+1.3282}{2} = 1.3263$

Now $f(x_8) = f(1.3263) = +ve$

Hence the root lies between 1.3243 and 1.3263.

Let $x_9 = \frac{1.3243+1.3263}{2} = 1.3253$

Now $f(x_9) = f(1.3253) = +ve$

Hence the root lies between 1.3243 and 1.3253.

Let $x_{10} = \frac{1.3243+1.3253}{2} = 1.3248$

Now $f(x_{10}) = f(1.3248) = +ve$

Hence the root lies between 1.3243 and 1.3248.

Let $x_{11} = \frac{1.3243+1.3248}{2} = 1.3246$

Now $f(x_{11}) = f(1.3246) = -ve$

Hence the root lies between 1.3248 and 1.3246.

Let $x_{12} = \frac{1.3248+1.3246}{2} = 1.3246$

Therefore, the approximate root is 1.3246.

CONCLUSION

Therefore, the approximate positive root of $x^3 - x = 1$ is 1.3246.

ITERATION METHOD

AIM

To find the roots of a linear equation.

OBJECTIVE

In this section you will learn to

- Separate the interval in which the root of the equation lies
- Find the root of the equation using iteration method.

PROCEDURE

Step 1: Rearrange f(x) so that x is on the left-hand side of the equation:

$$x = \emptyset(x)$$

Step 2: If $f(x_0)$ and $f(x_1)$ are of opposite signs, at least one real root lie between x_0 and x_1 should exist.

PROBLEMS

Find the root of the equation $x = ½ + \sin x$, using the iteration method.

Solution:

Let $f(x) = \sin x - x + ½$

$f(1) = \sin 1 - 1 + ½ = 0.84 - 0.5 = +ve$

$f(2) = \sin 2 - 2 + ½ = 0.9 - 1.5 = -ve.$

A root lies between 1 and 2. The given equation can be written as

$x = \sin x + ½ = f(x)$

$|\phi'(x)| = |\cos x| < 1$ in $[1,2]$

Hence the iteration method can be applied.

Let the approximation be $x_0 = 1$. The successive approximation is as follows:

$x_1 = f(x_0) = \sin 1 + ½ = 0.8414 + 0.5 = 1.3414$

$x_2 = f(x_1) = \sin(1.3414) + ½ = 0.9738 + 0.5 = 1.4738$

$x_3 = f(x_2) = \sin(1.4738) + ½ = 0.9952 + 0.5 = 1.4952$

$x_4 = f(x_3) = \sin(1.4952) + ½ = 0.9971 + 0.5 = 1.4971$

$x_5 = f(x_4) = \sin(1.4971) + ½ = 0.9972 + 0.5 = 1.4972$

Since x_4 and x_5 are almost equal, the required root is 1.497.

CONCLUSION

Therefore, the root of the equation $x = ½ + \sin x$ is 1.497.

Unit-II

NEWTON'S INTERPOLATION FORMULA FOR FORWARD DIFFERENCE

AIM

To find the value of $y = f(x)$ between the range of values of x near the initial stage.

OBJECTIVE

In this section you will learn to

- Approximate the values of a function at points that lie between given data points

PROCEDURE

The Newton's forward interpolation formula is

$$y(x_0 + nh) = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \cdots$$

PROBLEMS

A function $f(x)$ is given by the following table. Find $f(0.2)$ by a suitable formula.

x	0	1	2	3	4	5	6
$f(x)$	176	185	194	203	212	220	229

Solution:

The difference table is follows

x	$y = f(x)$	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$	$\Delta^6 y_0$
0	176						
1	185	9					
2	194	9	0				
3	203	9	0	0			
4	212	9	0	0	0		
5	220	8	-1	-1	-1	-1	
6	229	9	1	2	3	4	5

Here $x_0 = 0$, $h = 1$, $y_0 = 176 = f(x)$

We have to find the value of $f(0.2)$. By Newton's forward interpolation formula, we have

$$y(x_0 + nh) = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \cdots$$

$x_0 + nh = 0.2$

$0 + n = 0.2 \Rightarrow n = 0.2$

$$f(0.2) = 176 + (0.2)9 + \frac{(0.2)(0.2-1)}{2}(0)$$

$$= 176 + 1.8$$

$$= 177.8$$

CONCLUSION

Hence, $f(0.2) = 177.8$

NEWTON'S INTERPOLATION FORMULA FOR BACKWARD DIFFERENCE

AIM

To find the value of $y = f(x)$ between the range of values of x near the lower end.

OBJECTIVE

In this section you will learn to

- Approximate the values of a function at points that lie between given data points

PROCEDURE

The Newton's backward interpolation formula is

$$y(x_0 + nh) = y_0 + n\nabla y_0 + \frac{n(n+1)}{2!}\nabla^2 y_0 + \cdots$$

PROBLEMS

From the given table compute the value of sin 38.

x	0	10	20	30	40
$\sin x$	0	0.17365	0.34202	0.5	0.64276

Solution:

As we have to determine the value of $y = \sin x$ near the lower end, we apply Newton's backward interpolation formula.

The difference table is as given below.

x_0	$y(x) = \sin x_0$	$\Delta \boldsymbol{y}$	$\Delta^2 \boldsymbol{y}$	$\Delta^3 \boldsymbol{y}$	$\Delta^4 \boldsymbol{y}$
0	0				
10	0.17365	0.17365			
20	0.34202	0.16837	- 0.00528		
30	0.5000	0.15798	- 0.01039	- 0.00511	
40	0.64279	0.14279	- 0.01519	- 0.0048	0.00031
x_0	y_0	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$

Here $x_0 = 40$, $h = 0.64279$, $h = 10$

Newton's backward differences formula

$$y(x_0 + nh) = y_0 + n\nabla y_0 + \frac{n(n+1)}{2!}\nabla^2 y_0 + \cdots$$

$x_0 + nh = 38 \Rightarrow 40 + n(10) = 38 \Rightarrow n = -0.2$

$y(38)$

$$= 0.64279 + (-0.2)(0.14279) + \frac{(-0.2)(-0.2+1)}{2!}(-0.01519)$$

$$+ \frac{(-0.2)(-0.2+1)(-0.2+2)}{3!}(-0.0048) + neglible\ term$$

$= 0.64279 - 0.028558 + 0.0012152 + 0.0002304$

$= 0.61566.$

CONCLUSION

Hence the value of $\sin 38$ is 0.61566.

Unit-III

LAGRANGE'S INTERPOLATION METHOD

AIM

To determine the function's value even when the parameters are not evenly spaced.

OBJECTIVE

In this section you will learn to

- Calculate the value of the independent variable x that corresponds to a given function value.

PROCEDURE

Step 1: Let $y = f(x)$ be a function which assumes the values $f(x_0), f(x_1) \ldots\ldots f(x_n)$ corresponding to the values $x_0, x_1, \ldots, x_n$.

$$y = f(x)$$

$$= \frac{(x - x_1)(x - x_2) \ldots (x - x_n)}{(x_0 - x_1)(x_0 - x_2) \ldots (x_0 - x_n)} f(x_0) + \frac{(x - x_0)(x - x_2) \ldots (x - x_n)}{(x_1 - x_0)(x_1 - x_2) \ldots (x_1 - x_n)} f(x_1) + \cdots$$

PROBLEMS

Using Lagrange's interpolation formula, find the value of y corresponding to x = 10 from the following table.

x	5	6	9	11
$f(x)$	12	13	14	16

Solution

We have $x_0 = 5,\ x_1 = 6, x_2 = 9,\ x_3 = 11$

$y_0 = 12, y_1 = 13, y_2 = 14, y_3 = 16$

Using Lagrange's interpolation formula, we have

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

Substitute

$$f(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)}(12) + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)}(13) + \frac{(10-5)(10-6)(10-11)}{(9-6)(9-6)(9-11)}14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \times 16$$

$$= 2 - \frac{13}{3} + \frac{35}{3} + \frac{16}{3} = \frac{42}{3}$$

CONCLUSION

Hence the value of y corresponding to $x = 10$ is $\frac{42}{3}$.

Unit-IV

GAUSS-ELIMINATION METHOD

AIM

To solve the system of equations.

OBJECTIVE

In this section you will learn to

- Use elementary row operations to put a matrix into row echelon form.
- Find the number of solutions in the system of equations.

PROCEDURE

Step 1. Locate the leftmost column that does not consist entirely of zeros.

Step 2. Interchange the top row with another row, if necessary, to bring a nonzero entry to the top of the column found in Step 1.

Step 3. If the entry that is now at the top of the column found in Step 1 is b , multiply the first row by b 1 in order to introduce a leading 1.

Step 4. Add suitable multiples of the top row to the rows below so that all entries below the leading 1 become zeros.

Step 5. Now cover the top row in the matrix and begin again with Step 1 applied to the submatrix that remains. Continue in this way until the entire matrix is in row - echelon form.

PROBLEMS

Solve the following equations by Gauss's Elimination method.

$$5x - y - 2z = 142$$

$$x - 3y - z = -30$$

$$2x - y - 3z = -5$$

Solution

The given system of equations are

$$5x - y - 2z = 142$$
$$x - 3y - z = -30$$
$$2x - y - 3z = -5$$

Now let us write these equations in matrix form

$$\left[\begin{array}{ccc|c} 5 & -1 & -2 & 142 \\ 1 & -3 & -1 & -30 \\ 2 & -1 & -3 & -5 \end{array}\right]$$

Now let us interchange the rows R_1 and R_2. i.e. $R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & -3 & -1 & -30 \\ 5 & -1 & -2 & 142 \\ 2 & -1 & -3 & -5 \end{array}\right]$$

Now let us make another operations as $R_2 \to R_2 - 5R_1$ and $R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & -3 & -1 & -30 \\ 0 & 14 & 3 & 292 \\ 0 & 5 & -1 & 55 \end{array}\right]$$

Now let us make another operations as $R_3 \to 14R_3 - 5R_2$

$$\left[\begin{array}{ccc|c} 1 & -3 & -1 & -30 \\ 0 & 14 & 3 & 292 \\ 0 & 0 & -29 & -690 \end{array}\right]$$

Here $x - 3y - z = -30, 14y + 3z = 292, -29z = -690.$

Thus $z = 23.79$

Substitute $z = 23.79$ in $14y + 3z = 292$, we get $y = 15.76$

Similarly, substitute $z = 23.79$ and $y = 15.76$ in $x - 3y - z = -30$, we get

$x = 41.07.$

Hence, $x = 41.07, y = 15.76$ and $z = 23.79$.

CONCLUSION

Hence, $x = 41.07, y = 15.76$ and $z = 23.79$.

GAUSS JORDAN ELIMINATION METHOD

AIM

To apply elementary row operations to a matrix until reduced row-echelon matrix is obtained.

OBJECTIVE

In this section you will learn to

- Solve linear systems using the Gauss-Jordan elimination method.
- Value the essence of cooperation in solving linear systems word problem.

PROCEDURE

Step 1. Write the augmented matrix of the system.

Step 2. Use row operations to transform the augmented matrix in the form described below, which is called the **reduced row echelon form** (RREF).

(a) The rows (if any) consisting entirely of zeros are grouped together at the bottom of the matrix.

(b) In each row that does not consist entirely of zeros, the leftmost non-zero element is a 1 (called a leading 1 or a pivot).

(c) Each column that contains a leading 1 has zeros in all other entries.

(d) The leading 1 in any row is to the left of any leading 1's in the rows below it.

Step 3. Stop process in step 2 if you obtain a row whose elements are all zeros except the last one on the right. In that case, the system is inconsistent and has no solutions. Otherwise, finish step 2 and read the solutions of the system from the final matrix.

PROBLEMS

Apply Jordan's method to solve

$$x + y + z = 1$$
$$4x + 3y - z = 6$$
$$3x + 5y + 3z = 4$$

Solution

The given system of equations are

$$x + y + z = 1$$
$$4x + 3y - z = 6$$
$$3x + 5y + 3z = 4$$

Using the augmented matrix $[A|I]$, we obtain

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 4 & 3 & -1 & 0 & 1 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{array}\right]$$

Now let us interchange the rows R_1 and R_2. i.e. $R_1 \leftrightarrow R_2$, we get

$$\sim \left[\begin{array}{ccc|ccc} 4 & 3 & -1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{array}\right]$$

Now let us multiply R_1 by $\frac{1}{4}$. i.e. $R_1 \to \frac{1}{4}R_1$, we get

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3/4 & -1/4 & 0 & 1/4 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 5 & 3 & 0 & 0 & 1 \end{array}\right]$$

Now let us make another operations as $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3/4 & -1/4 & 0 & 1/4 & 0 \\ 0 & 1/4 & 5/4 & 1 & -1/4 & 0 \\ 0 & 11/4 & 15/4 & 0 & -\frac{3}{4} & 1 \end{array}\right]$$

Now let us interchange the rows R_2 and R_3. i.e. $R_2 \leftrightarrow R_3$, we get

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3/4 & -1/4 & 0 & 1/4 & 0 \\ 0 & 11/4 & 15/4 & 0 & -3/4 & 1 \\ 0 & 1/4 & 5/4 & 1 & -1/4 & 0 \end{array}\right]$$

Now let us multiply R_2 by $\frac{4}{11}$. i.e. $R_2 \rightarrow \frac{4}{11}R_2$, we get

$$\sim \left[\begin{array}{ccc|ccc} 1 & 3/4 & -1/4 & 0 & 1/4 & 0 \\ 0 & 1 & 15/11 & 0 & -3/11 & 4/11 \\ 0 & 1/4 & 5/4 & 1 & -1/4 & 0 \end{array}\right]$$

Now let us make another operations as $R_1 \rightarrow R_1 - (3/4)R_2$ and $R_3 \rightarrow R_3 - (1/4)R_2$, we get

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -14/11 & 0 & 5/11 & -3/11 \\ 0 & 1 & 15/11 & 0 & -3/11 & 4/11 \\ 0 & 0 & 10/11 & 1 & -2/11 & -1/11 \end{array}\right]$$

Now let us multiply R_3 by $\frac{11}{10}$. i.e. $R_3 \rightarrow \frac{11}{10}R_3$, we get

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -14/11 & 0 & 5/11 & -3/11 \\ 0 & 1 & 15/11 & 0 & -3/11 & 4/11 \\ 0 & 0 & 1 & 11/10 & -1/5 & -1/10 \end{array}\right]$$

Now let us make another operations as $R_1 \rightarrow R_1 + R_3$ and $R_2 \rightarrow R_2 - 2R_3$, we get

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/5 & 1/5 & -2/5 \\ 0 & 1 & 0 & -3/2 & 0 & 1/2 \\ 0 & 0 & 1 & 11/10 & -1/5 & -1/10 \end{array}\right]$$

Therefore, the solution of the system is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7/5 & 1/5 & -2/5 \\ -3/2 & 0 & 1/2 \\ 11/10 & -1/5 & -1/10 \end{bmatrix} \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1/2 \\ -1/2 \end{bmatrix}$$

CONCLUSION

Hence, the values of x_1, x_2 and x_3 are 1, $\frac{1}{2}$ and $-\frac{1}{2}$ respectively.

Unit-V

GAUSS JACOBI METHOD

AIM

To find the solution for the system of linear equations.

OBJECTIVE

In this section you will learn to

- To determine solution of a set of linear equation using Gauss-Jacobi method.
- To understand the convergence property of the method.
- To study the effect of using different initial values and error tolerance on the solution.

PROCEDURE

Step 1: First check for convergence of approximation.

Step 2: Take the initial approximation, $x_1^{(0)} = 0, x_2^{(0)} = 0$ and $x_3^{(0)} = 0$.

Step 3: To estimate the value of x, $x^{(k+1)} = D^{-1}\left(b - Rx^{(k)}\right)$.

PROBLEM

Solve the system of equations using the Jacobi method

$$26x_1 + 2x_2 + 2x_3 = 12.6$$

$$3x_1 + 27x_2 + x_3 = -14.3$$

$$2x_1 + 3x_2 + 17x_3 = 6.0$$

Obtain the result correct to three decimal places.

Solution

First, check for the convergence of approximations,

$$26 > 2 + 2$$

$$27 > 3 + 1$$

$$17 > 2 + 3$$

Hence, the given system of equations are strongly diagonally dominant, which ensures the convergence of approximations. Let us take the initial approximation, $x_1^{(0)} = 0, x_2^{(0)} = 0$ and $x_3^{(0)} = 0$.

First iteration:

$$x_1^{(1)} = \frac{1}{26}[12.6 - 2(0) - 2(0)] = 0.48462$$

$$x_2^{(1)} = \frac{1}{27}[-14.3 - 3(0) - 1(0)] = -0.52963$$

$$x_3^{(1)} = \frac{1}{17}[6 - 2(0) - 3(0)] = 0.35294$$

Second iteration:

$$x_1^{(2)} = \frac{1}{26}[12.6 - 2(-0.52963) - 2(0.35294)] = 0.49821$$

$$x_2^{(2)} = \frac{1}{27}[-14.3 - 3(0.48462) - 1(0.35294)] = -0.59655$$

$$x_3^{(2)} = \frac{1}{17}[6 - 2(0.48462) - 3(-0.52963)] = 0.38939$$

Third iteration:

$$x_1^{(3)} = \frac{1}{26}[12.6 - 2(-0.59655) - 2(0.38939)] = 0.50055$$

$$x_2^{(3)} = \frac{1}{27}[-14.3 - 3(0.49821) - 1(0.38939)] = -0.59941$$

$$x_3^{(3)} = \frac{1}{17}[6 - 2(0.49821) - 3(-0.59655)] = 0.39960$$

Fourth iteration:

$$x_1^{(4)} = \frac{1}{26}[12.6 - 2(-0.59941) - 2(0.39960)] = 0.49999$$

$$x_2^{(4)} = \frac{1}{27}[-14.3 - 3(0.50055) - 1(0.39960)] = -0.60005$$

$$x_3^{(4)} = \frac{1}{17}[6 - 2(0.50055) - 3(-0.59941)] = 0.39983$$

Fifth iteration:

$$x_1^{(5)} = \frac{1}{26}[12.6 - 2(-0.60005) - 2(0.39983)] = 0.50002$$

$$x_2^{(5)} = \frac{1}{27}[-14.3 - 3(0.49999) - 1(0.39983)] = -0.59999$$

$$x_3^{(5)} = \frac{1}{17}[6 - 2(0.49999) - 3(-0.60005)] = 0.40001$$

Sixth iteration:

$$x_1^{(5)} = \frac{1}{26}[12.6 - 2(-0.59999) - 2(0.40001)] = 0.50000$$

$$x_2^{(5)} = \frac{1}{27}[-14.3 - 3(0.50002) - 1(0.40001)] = -0.60000$$

$$x_3^{(5)} = \frac{1}{17}[6 - 2(0.50002) - 3(-0.59999)] = 0.40000$$

After the fifth iteration, we get

$\left|x_1^{(5)} - x_1^{(4)}\right| = |0.49999 - 0.50000| = 0.00001$

$\left|x_2^{(5)} - x_2^{(4)}\right| = |-0.60005 + 0.60000| = 0.00005$

$\left|x_3^{(5)} - x_3^{(4)}\right| = |0.39983 - 0.40000| = 0.00017$

Since, all the errors in magnitude are less than 0.0005, the required solution is

$x_1 = 0.5, x_2 = -0.6, x_3 = 0.4$

CONCLUSION

Hence, the values of x_1, x_2 and x_3 are 0.5, -0.6 and 0.4 respectively.

GAUSS SEIDEL METHOD

AIM

To find the solution for the system of linear equations.

OBJECTIVE

In this section you will learn to

- To determine solution of a set of linear equation using Gauss-Seidel method.
- To understand the convergence property of the method.
- To study the effect of using different initial values and error tolerance on the solution.

PROCEDURE

Step 1: First check for convergence of approximation.

Step 2: Take the initial approximation, $x_1^{(0)} = 0, x_2^{(0)} = 0$ and $x_3^{(0)} = 0$.

Step 3: Use new values of $x_i^{(k+1)}$ as soon as they are known.

PROBLEM

Solve the system of equations using the Gauss-Seidel method

$$45x_1 + 2x_2 + 3x_3 = 58$$
$$-3x_1 + 22x_2 + 2x_3 = 47$$
$$5x_1 + x_2 + 20x_3 = 67$$

Obtain the result correct to three decimal places.

Solution

First, check for the convergence of approximations,

$45 > 2 + 3$

$22 > -3 + 2$

$20 > 5 + 1$

Hence, the given system of equations are strongly diagonally dominant, which ensures the convergence of approximations. Let us take the initial approximation, $x_1^{(0)} = 0, x_2^{(0)} = 0$ and $x_3^{(0)} = 0$.

First iteration:

$$x_1^{(1)} = \frac{1}{45}[58 - 2(0) - 3(0)] = 1.28889$$

$$x_2^{(1)} = \frac{1}{22}[47 + 3(1.28889) - 2(0)] = 2.31212$$

$$x_3^{(1)} = \frac{1}{20}[67 - 5(1.28889) - 1(2.31212)] = 2.91217$$

Second iteration:

$$x_1^{(2)} = \frac{1}{45}[58 - 2(2.31212) - 3(2.91217)] = 0.99198$$

$$x_2^{(2)} = \frac{1}{22}[47 + 3(0.99198) - 2(2.91217)] = 2.00689$$

$$x_3^{(2)} = \frac{1}{20}[67 - 5(0.99198) - 1(2.00689)] = 3.00166$$

Third iteration:

$$x_1^{(3)} = \frac{1}{45}[58 - 2(2.00689) - 3(3.00166)] = 0.99958$$

$$x_2^{(3)} = \frac{1}{22}[47 + 3(0.99958) - 2(3.00166)] = 1.99979$$

$$x_3^{(3)} = \frac{1}{20}[67 - 5(0.99958) - 1(1.99979)] = 3.00012$$

Fourth iteration:

$$x_1^{(4)} = \frac{1}{45}[58 - 2(1.99979) - 3(3.00012)] = 1.00000$$

$$x_2^{(4)} = \frac{1}{22}[47 + 3(1.00000) - 2(3.00012)] = 1.99998$$

$$x_3^{(4)} = \frac{1}{20}[67 - 5(1.00000) - 1(1.99998)] = 3.00000$$

After the fourth iteration, we get

$$\left|x_1^{(4)} - x_1^{(3)}\right| = |1.00000 - 0.99958| = 0.00042$$

$$\left|x_2^{(4)} - x_2^{(3)}\right| = |1.99999 - 1.99979| = 0.00020$$

$$\left|x_3^{(4)} - x_3^{(3)}\right| = |3.00000 - 3.00012| = 0.00012$$

Since, all the errors in magnitude are less than 0.0005, the required solution is

$$x_1 = 1.0, x_2 = 1.99999, x_3 = 3.0$$

Rounding to three decimal places, we get

$$x_1 = 1.0, x_2 = 2.0, x_3 = 3.0$$

CONCLUSION

Hence, the values of x_1, x_2 and x_3 are 1.0, 2.0 and 3.0 respectively.

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